2-3

Additional assumptions —

(i) Density of liquid, ρ, and density of coolant, ρ_J, are constant.

(ii) Specific heats of the liquid, C, and of the coolant, C_J, are constant

Since V is constant, mass balance for the tank gives

$$\rho \frac{dV}{dt} = q_F - q = 0 \quad ; \text{ thus } \quad q = q_F \, .$$

Energy balance for tank is

$$\rho V C \frac{dT}{dt} = q_F \rho C (T_F - T) - K q_J^{0.8} A (T - T_J) \qquad (1)$$

Energy balance for the jacket is

$$\rho_J V_J C_J \frac{dT_J}{dt} = q_J \rho_J C_J (T_i - T_J) + K q_J^{0.8} A (T - T_J) \qquad (2)$$

where A is the heat transfer area in ft^2 between the process liquid and the coolant.

Equations (1) and (2) comprise the dynamic model for the system.

2-4

Additional assumptions —

(i) The density, ρ, and the specific heat, C, of the process liquid are constant.

(ii) The temperature of steam, T_s, is uniform over the entire heat transfer area.

(iii) T_s is a function of P_s, $T_s = f(P_s)$

Mass balance for the tank is

$$\frac{dV}{dt} = q_F - q \qquad (1)$$

Energy balance for the tank is

$$\rho C \frac{d[V(T-T_{ref})]}{dt} = q_F \rho C (T_F - T_{ref}) - q \rho C (T - T_{ref}) + UA(T_s - T) \qquad (2)$$

where T_{ref} is a constant reference temperature
A is the heat transfer area

Equation (2) is simplified by substituting for $\frac{dV}{dt}$ from equation (1), and replacing T_s by $f(P_s)$, to give

$$\rho C V \frac{dT}{dt} = q_F \rho C (T_F - T) + UA[f(P_s) - T] \qquad (3)$$

Then, equations (1) and (3) constitute the dynamic model for the system.

$\boxed{2-7}$

(a) For linear valve flow characteristics, the mass flows are

$$w_a = \frac{P_d - P_1}{R_a}, \quad w_b = \frac{P_1 - P_2}{R_b}, \quad w_c = \frac{P_2 - P_f}{R_c} \qquad (1)$$

Material balance for the surge tanks —

$$\frac{dm_1}{dt} = w_a - w_b, \qquad \frac{dm_2}{dt} = w_b - w_c \qquad (2)$$

where m_1, m_2 are masses of gas in the surge tanks.

If ideal gas law holds, then

$$P_1 V_1 = \frac{m_1}{M} RT_1, \qquad P_2 V_2 = \frac{m_2}{M} RT_2 \qquad (3)$$

where M is the molecular weight of the gas

T_1, T_2 are the temperatures in the surge tanks.

Substituting for m_1 and m_2 from equation (3) into equation (2), and noticing that V_1, T_1, V_2, T_2 are constant,

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = w_a - w_b, \qquad \frac{V_2 M}{RT_2}\frac{dP_2}{dt} = w_b - w_c \qquad (4)$$

The dynamic model consists of equations (1) and (4).

2-5

Assume constant liquid density, ρ.

Mass balance for the tank is

$$\frac{d(\rho A h + m_g)}{dt} = \rho (q_i - q)$$

Since ρ, A, and m_g are constant, this becomes

$$A \frac{dh}{dt} = q_i - q \qquad (1)$$

The square-root relationship for flow through valve is

$$q = C_v \left(P_g + \frac{\rho g h}{g_c} - P_a \right)^{1/2} \qquad (2)$$

From ideal gas law

$$P_g = \frac{(m_g/M) RT}{A(H-h)} \qquad (3)$$

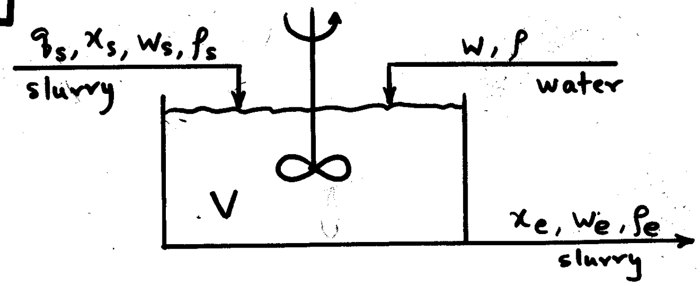
Where T is the absolute temperature of the gas.

Equation (1) gives the unsteady-state model upon substitution of q from equation (2) and of P_g from eq. (3),

$$A \frac{dh}{dt} = q_i - C_v \left[\frac{(m_g/M)RT}{A(H-h)} + \frac{\rho g h}{g_c} - P_a \right]^{1/2} \qquad (4)$$

Since the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and equation (2) shows that system is independent of P_a.

2-9



Assume 1) perfect mixing in the tank
2) no volume changes on mixing

Let ρ be the density of pure water, assumed constant
ρ_t be the density of pure solid, assumed constant
ρ_s, ρ_e be the slurry densities in inlet and exit streams, resp.
W_s, W_e be the mass flow rate of inlet and exit slurries, resp.

Then, by definition, $W_s = \rho_s q_s$ (1)

$$\rho_s = \frac{W_s}{\left(\frac{x_s W_s}{\rho_t} + \frac{(1-x_s)W_s}{\rho}\right)} = \frac{1}{\left(\frac{x_s}{\rho_t} + \frac{1-x_s}{\rho}\right)} \quad (2)$$

$$\rho_e = \frac{1}{\left(\frac{x_e}{\rho_t} + \frac{1-x_e}{\rho}\right)} \quad (3)$$

Since volume, V, is constant, an overall material balance gives

$$V\frac{d\rho_e}{dt} = W_s + W - W_e$$

and a component balance for solids gives

$$V\frac{d(\rho_e x_e)}{dt} = W_s x_s - W_e x_e$$

or $\rho_e V \frac{dx_e}{dt} = -x_e V\frac{d\rho_e}{dt} + W_s x_s - W_e x_e = -x_e(W_s + W - W_e) + W_s x_s - W_e x_e$

Substituting for ρ_e, ρ_s, W_s from equations (1)-(3), gives the model

$$\frac{1}{\left(\frac{x_e}{\rho_t} + \frac{1-x_e}{\rho}\right)} V\frac{dx_e}{dt} = \frac{1}{\left(\frac{x_s}{\rho_t} + \frac{1-x_s}{\rho}\right)} q_s (x_s - x_e) - W x_e$$

2-10

Let C_{Ai}, C_{Bi} be molar concentrations (mole/ft^3) of A and B in the feed, respectively,

C_A, C_B, C_c be molar concentrations (mole/ft^3) of A, B, and C in the exit stream, resp.,

M_A, M_B be molecular weights of A and B, resp.,

ρ_A, ρ_B be densities of A and B, resp.

An overall material balance over the CSTR indicates that the flow rate of exit stream = q, since V is constant. Since feed contains only A and B, the flow rate q must equal the sum of volumetric flow rates of A and B in feed,

$$q = q C_{Ai} \frac{M_A}{\rho_A} + q C_{Bi} \frac{M_B}{\rho_B}$$

or
$$C_{Ai} = \frac{\rho_A}{M_A}\left[1 - C_{Bi}\frac{M_B}{\rho_B}\right] \qquad (1)$$

Component balances over the CSTR for A, B, and C yield

$$V\frac{dC_A}{dt} = qC_{Ai} - qC_A - V(k_1 C_A^2 + k_2 C_A C_B) \qquad (2)$$

$$V\frac{dC_B}{dt} = qC_{Bi} - qC_B + V(k_1 C_A^2 - k_2 C_A C_B) \qquad (3)$$

$$V\frac{dC_c}{dt} = -qC_c + Vk_2 C_A C_B \qquad (4)$$

Substituting for C_{Ai} from equation (1) into equation (2) gives

$$V\frac{dC_A}{dt} = q\frac{\rho_A}{M_A}\left[1 - C_{Bi}\frac{M_B}{\rho_B}\right] - qC_A - V(k_1 C_A^2 + k_2 C_A C_B) \qquad (5)$$

Since k_1, k_2 are constant for isothermal reactor, equations (3), (4), and (5) relate C_A, C_B, C_c to variations in C_{Bi} and q. These equations constitute the unsteady-state model.

3-19

(a) Assume

1) The freezer is isothermal
2) The temperature of the beer is uniform over the can.
3) The beer can is a standard $2\frac{1}{2}$" OD, 6" tall monster with a surface area of 0.327 ft^2.
4) Heat transfer occurs only in the air around the can by means of natural convection, with a heat tranfer coefficient of 2.2 $\frac{Btu}{hr\,°F\,ft^2}$ (ref. - Perry's handbook).

Energy balance over the beer,

$$mC\frac{dT_b}{dt} = UA\left(T_s - T_b\right) \qquad (1)$$

where
m is mass of beer (0.8 lb)
C is specific heat of beer (~1 Btu/lb °F)
T_b is the beer temperature
U is heat transfer coefficient (2.2 Btu/hr°F ft^2)
A is heat transfer area (0.327 ft^2)
T_s is temperature of the surroundings.

Taking Laplace transform of eq. (1) written in deviation variables,

$$s\tilde{T}_b(s) - \tilde{T}_b(0) = \frac{UA}{mC}\left(\tilde{T}_s(s) - \tilde{T}_b(s)\right)$$

$$s\tilde{T}_b(s) - 0 = 1.112\left(\frac{-6-85}{s} - \tilde{T}_b(s)\right)$$

$$\tilde{T}_b(s) = \frac{-91\times 1.112}{s(s+1.112)} = -\frac{91}{s} + \frac{91}{s+1.112}$$

$$\tilde{T}_b(t) = -91\left(1-e^{-1.112t}\right), \quad t = -\frac{1}{1.112}\ln\left[1 - \frac{T_b(t)-85}{-91}\right]$$

for $T_b(t) = 60$, $t = 0.357$ hr.

(b) Lower T_s or higher U are desirable. Moving the can around in an ice bath may increase U enough to give faster cooling even with a higher T_s.

5-8

(a) Assume that q is constant.

Material balance over the tank,

$$A\frac{dh}{dt} = q_1 + q_2 - q$$

Writing in deviation variables and taking Laplace transform

$$AsH'(s) = Q_1'(s) + Q_2'(s)$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{1}{As}$$

(b) $q_1'(t) = 5\,S(t) - 5\,S(t-12)$

$$Q_1'(s) = \frac{5}{s} - \frac{5}{s}e^{-12s}$$

$$H'(s) = \frac{1}{As}Q_1'(s) = \frac{5/A}{s^2} - \frac{5/A}{s^2}e^{-12s}$$

$$h'(t) = \frac{5}{A}t\,S(t) - \frac{5}{A}(t-12)\,S(t-12)$$

$$h(t) = \begin{cases} 4 + \frac{5}{A}t = 4 + 0.177t & 0 \le t \le 12 \\ 4 + (\frac{5}{A}\times 12) = 6.122 & 12 < t \end{cases}$$

(c) $\bar{h} = 6.122$ ft at the new steady state $t \geq 12$.

(d) $q_1'(t) = 5\,S(t) - 10\,S(t-12) + 5\,S(t-24)$; $t_w = 12$

$$Q_1'(s) = \frac{5}{s}\left(1 - 2e^{-12s} + e^{-24s}\right)$$

$$H'(s) = \frac{5/A}{s^2} - \frac{10/A}{s^2}e^{-12s} + \frac{5/A}{s^2}e^{-24s}$$

$$h(t) = 4 + 0.177t\,S(t) - 0.354(t-12)S(t-12) + 0.177(t-24)\,S(t-24)$$

$$\bar{h} = 4 + 0.177t - 0.354(t-12) + 0.177(t-24) = 4 \text{ ft at } t \geq 24$$

5-19

(a) For the original system,

$$A_1 \frac{dh_1}{dt} = C q_i - \frac{h_1}{R_1}$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

where $A_1 = A_2 = \pi \times (3)^2/4 = 7.07 \text{ ft}^2$

$$C = 0.1337 \ \frac{\text{ft}^3/\text{min}}{\text{gpm}}$$

$$R_1 = R_2 = \frac{\bar{h}_1}{C\bar{q}_i} = \frac{2.5}{0.1337 \times 100} = 0.187 \frac{\text{ft}}{\text{ft}^3/\text{min}}$$

Using deviation variables and taking laplace transforms,

$$\frac{H_1'(s)}{Q_i'(s)} = \frac{C}{A_1 s + \frac{1}{R_1}} = \frac{CR_1}{A_1 R_1 s + 1} = \frac{0.025}{1.32s+1}$$

$$\frac{H_2'(s)}{H_1'(s)} = \frac{1/R_1}{A_2 s + \frac{1}{R_2}} = \frac{R_2/R_1}{A_2 R_2 s + 1} = \frac{1}{1.32s+1}$$

$$\frac{H_2'(s)}{Q_i'(s)} = \frac{0.025}{(1.32s+1)^2}$$

For step change in q_i of magnitude M,

$h'_{1\,max} = 0.025\,M$; and

$h'_{2\,max} = 0.025\,M$ since the second-order transfer function $\frac{0.025}{(1.32s+1)^2}$ is critically damped $(\zeta = 1)$

Hence $M_{max} = \dfrac{2.5 \text{ ft}}{0.025 \text{ ft/gpm}} = 100 \text{ gpm}$

For the modified system,

$$A\frac{dh}{dt} = Cq_i - \frac{h}{R}$$

$$A = \pi \times (4)^2/4 = 12.6 \text{ ft}^2$$

$$V = V_1 + V_2 = 2 \times 7.07 \text{ ft}^2 \times 5 \text{ ft} = 70.7 \text{ ft}^3$$

$$h_{max} = V/A = 5.62 \text{ ft}$$

$$R = \frac{\bar{h}}{C\bar{q}_i} = \frac{0.5\, h_{max}}{C\,\bar{q}_i} = \frac{0.5 \times 5.62}{0.1337 \times 100} = 0.21 \frac{\text{ft}}{\text{ft}^3/\text{min}}$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{C}{As + \frac{1}{R}} = \frac{CR}{ARs+1} = \frac{0.0281}{2.64s+1}$$

$$h'_{max} = 0.0281\, M \quad , \quad M_{max} = \frac{2.81 \text{ ft}}{0.0281 \frac{\text{ft}}{\text{gpm}}} = 100 \text{ gpm}$$

Hence, both systems can handle the same maximum step disturbance in q_i.

(b) For step change of magnitude M, $Q_i'(s) = M/s$

For original system,

$$Q_2'(s) = \frac{1}{R_2}H_2'(s) = \frac{1}{0.187}\,\frac{0.025}{(1.32s+1)^2}\,\frac{M}{s}$$

$$= 0.134M\left[\frac{1}{s} - \frac{1.32}{(1.32s+1)} - \frac{1.32}{(1.32s+1)^2}\right]$$

$$q_2'(t) = 0.134M\left[1 - \left(1 + \frac{t}{1.32}\right)e^{-t/1.32}\right]$$

For modified system,

$$Q'(s) = \frac{1}{R}H'(s) = \frac{1}{0.21}\,\frac{0.0281}{(2.64s+1)}\,\frac{M}{s} = 0.134M\left[\frac{1}{s} - \frac{2.64}{2.64s+1}\right]$$

$$q'(t) = 0.134M\left[1 - e^{-t/2.64}\right]$$

Original system provides better damping since $q_2'(t) < q'(t)$ for $t < 3.4$

3-17

(a) Assume perfect mixing in the mix tank. Overall material balance indicates that the flow rate of solution leaving the mix tank is equal to $q_c + q_w$, since the volume is constant and the density of caustic solution is the same as that of water.

Component balance for the caustic over the tank,

$$V \frac{dc}{dt} = q_c c_c - (q_c + q_w) c \quad , \qquad c(0) = 0$$

(b) $c_c >> c$ implies $c_c - c \simeq c_c$

$$V \frac{dc}{dt} = q_c (c_c - c) - q_w c \simeq q_c c_c - q_w c \; , \quad c(0) = 0$$

This condition will occur for small t, large V, and $q_c << q_w$.

(c) Taking Laplace transform of the model equation in (b),

$$V s C(s) = q_c \frac{c_c}{s} - q_w C(s)$$

where it is assumed that c_c is constant

$$C(s) = \frac{q_c c_c / V}{s(s + q_w/V)} = \frac{\alpha_1}{s} + \frac{\alpha_2}{(s + q_w/V)}$$

$$\alpha_1 = \left. \frac{q_c c_c / V}{(s + q_w/V)} \right|_{s=0} = \frac{q_c c_c}{q_w} \; , \quad \alpha_2 = \left. \frac{q_c c_c / V}{s} \right|_{s = -q_w/V} = - \frac{q_c c_c}{q_w}$$

$$C(s) = \frac{q_c c_c}{q_w} \left[\frac{1}{s} - \frac{1}{s + q_w/V} \right]$$

$$c(t) = \frac{q_c c_c}{q_w} \left[1 - e^{-(q_w/V)t} \right]$$