

Input Change

$$F_{in}^{spec}(0) = 1.2F_{in}(0)$$

The independent variables are time and F_{in}^{spec} , the dependent variables are F_{in} , H , and H_s , and the parameters are the cross-sectional area of the tank, the density of the fluid in the tank, k , and τ_v .

3.29 The model equations for this system are

Actuator system
$$\frac{dF_3}{dt} = \frac{1}{\tau_v}(F_3^{spec} - F_3)$$

Process model
$$\rho A_1 \frac{dH_1}{dt} = F_1 - k\sqrt{H_1}$$

$$\rho A_2 \frac{dH_2}{dt} = k\sqrt{H_1} - F_3$$

Sensor

$$H_{s,1} = H_1$$

$$H_{s,2} = H_2$$

The dependent variables are H_1 , H_2 , $H_{s,1}$, $H_{s,2}$, and F_3 , and the independent variables are time and F_3^{spec} , and the parameters are ρ , A_1 , A_2 , k , and τ_v .

3.30 The model equations for this system are

Actuator system
$$\frac{dF_1}{dt} = \frac{1}{\tau_v}(F_1^{spec} - F_1)$$

Process model
$$\rho A_1 \frac{dH_1}{dt} = F_1 - k_1\sqrt{H_1 + H_D} - H_2$$

$$\rho A_2 \frac{dH_2}{dt} = k_1\sqrt{H_1 + H_D} - H_2 - k_2\sqrt{H_2}$$

Sensor

$$H_{s,1} = H_1$$

$$H_{s,2} = H_2$$

The dependent variables are H_1 , H_2 , $H_{s,1}$, $H_{s,2}$, and F_1 , and the independent variables are time and F_1^{spec} , and the parameters are ρ , A_1 , A_2 , k_1 , k_2 , and τ_v .

3.31 The model equations for this system are

$$\begin{aligned}
 \text{Actuator system} \quad & \frac{dF_F}{dt} = \frac{1}{\tau_v} (F_F^{spec} - F_F) \\
 & \frac{dF_R}{dt} = \frac{1}{\tau_v} (F_R^{spec} - F_R) \\
 \\
 \text{Process model} \quad & V \frac{dC_{A_1}}{dt} = F_F C_{A_0} + F_R C_{A_2} - F_F C_{A_1} \\
 & V \frac{dC_{A_2}}{dt} = F_R (C_{A_1} - C_{A_2}) \\
 & F_I = F_F + F_R \\
 & F_P = F_R \\
 \\
 \text{Sensor} \quad & C_A(t) = C_{A_2}(t-3)
 \end{aligned}$$

The dependent variables are F_R , F_F , C_{A_1} , C_{A_2} , and C_A , and the independent variables are F_R^{spec} , F_F^{spec} , and time, and the parameters are V , the analyzer delay and τ_v .

3.32 The equations Example 3.4 of the text can be modified to model a stirred tank heater. Note that since no reactions take place, the composition of C_A does not require modeling.

$$\text{Lumped Actuator} \quad \frac{dQ}{dt} = \frac{1}{\tau_H} (Q_{spec} - Q)$$

$$\text{Process} \quad V_r \rho C_p \frac{dT}{dt} = F C_p (T_0 - T) + Q \quad (\text{Energy balance around stirred tank heater})$$

$$\text{Sensor} \quad \frac{dT_s}{dt} = \frac{1}{\tau_{Ts}} (T - T_s)$$

where Q is the actual heat addition rate.

t is the time.

ρ is the density of the liquid in the stirred tank heater.

Q_{spec} is the specified heat addition rate.

τ_H is the time constant for the heat addition to the stirred tank mixer.

V_r is the volume of the stirred tank heater.

C_p is the heat capacity of the liquid in the stirred tank heater.

T is the temperature of the liquid in the stirred tank heater.

F is the mass feed rate to the stirred tank heater.

T_0 is the temperature of the feed to the stirred tank heater.

T_s is the measured temperature of liquid in the stirred tank heater.

τ_{Ts} is the time constant of the temperature sensor.

Q_{spec} , T_0 , F and time are the independent variables. Q , T , and T_s are the dependent variables. V_r , ρ , τ_H , and C_p are parameters.

3.33 Since this reactor is isothermal, no energy balance is necessary. Dynamic mole balances for component A and B are required along with the model for the flow control loop on the feed stream and the model for the composition analyzer for measuring the composition of B in the product stream. Note that since F is in kg/s, the density is required for the convective term.

Actuator
$$\frac{dF}{dt} = \frac{1}{\tau_v} (F_{spec} - F)$$

Process
$$V_r \frac{dC_A}{dt} = \frac{F}{\rho} (C_{A0} - C_A) - V_r k_1 C_A \quad (\text{mole balance on A})$$

$$V_r \frac{dC_B}{dt} = -\frac{F}{\rho} C_B + V_r k_1 C_A - V_r k_2 C_B \quad (\text{mole balance on B})$$

Sensor
$$C_{Bs}(t) = C_B(t - \theta_A)$$

where F is the mass feed rate to the reactor (kg/s).

t is time (s).

τ_v is the time constant for the flow control loop on the feed stream (2.0 s).

F_{spec} is the specified mass flow rate to the reactor (kg/s).

V_r is the volume of the stirred tank heater (l).

C_A is the reactant concentration in the reactor (gmoles/l).

C_{A0} is the reactant concentration in the feed to the reactor (gmoles/l).

ρ is the density of the reactor feed and product (kg/l).

k_1 is the rate constant for the first reaction (s)⁻¹.

k_2 is the rate constant for the second reaction (s)⁻¹.

C_B is the concentration of product B in the reactor (gmoles/l).

C_{Bs} is the measured concentration of product B in the reactor (gmoles/l).

θ_A is the analyzer deadtime for measuring C_B (180 s)

F_{spec} , C_{A0} , and time are the independent variables. F , C_A , C_B , and C_{Bs} are the dependent variables. V_r , ρ , τ_v , k_1 , k_2 , and θ_A are the parameters.

3.34 Now Arrhenius rate constants are used and an energy balance must be added to the model for the nonisothermal CSTR.

$$\text{Actuators} \quad \frac{dF}{dt} = \frac{1}{\tau_v} (F_{spec} - F)$$

$$\frac{dQ}{dt} = \frac{1}{\tau_H} (Q_{spec} - Q)$$

$$\text{Process} \quad V_r \frac{dC_A}{dt} = \frac{F}{\rho} (C_{A0} - C_A) - V_r k_1 e^{-E_1/RT} C_A \quad (\text{mole balance on A})$$

$$V_r \frac{dC_B}{dt} = -\frac{F}{\rho} C_B + V_r k_1 e^{-E_1/RT} C_A - V_r k_2 e^{-E_2/RT} C_B \quad (\text{mole balance on B})$$

$$V_r \rho C_v \frac{dT}{dt} = F C_p (T_o - T) - V_r \Delta H_1 C_A k_1 e^{-E_1/RT} - V_r \Delta H_2 C_B k_2 e^{-E_2/RT} + Q$$

(Energy balance around reactor)

$$\text{Sensors} \quad C_{Bs}(t) = C_B(t - \theta_A)$$

$$\frac{dT_s}{dt} = \frac{1}{\tau_{Ts}} (T - T_s)$$

where F is the mass feed rate to the reactor (kg/s).

t is time (s).

τ_v is the time constant for the flow control loop on the feed stream (2.0 s).

F_{spec} is the specified mass flow rate to the reactor (kg/s).

V_r is the volume of the stirred tank heater (l).

C_A is the reactant concentration in the reactor (gmoles/l).

C_{A0} is the reactant concentration in the feed to the reactor (gmoles/l).

ρ is the density of the reactor feed and product (kg/l).

k_1 is the rate constant for the first reaction (s).

k_2 is the rate constant for the second reaction (s).

C_B is the concentration of product B in the reactor (gmoles/l).

C_{Bs} is the measured concentration of product B in the reactor (gmoles/l).

θ_A is the analyzer deadtime for measuring C_B (300 s)

Q is the actual heat addition rate (cal/s).

Q_{spec} is the specified heat addition rate (cal/s).

τ_H is the time constant for the heat addition to the CSTR (6 s).

T_o is the temperature of the feed to the stirred tank heater (K).

T_s is the measured temperature of liquid in the stirred tank heater (K).

R is the gas constant (cal/gmole-K).

C_p is the heat capacity of the liquid in the reactor and is assumed equal to C_v .

τ_{Ts} is the time constant of the temperature sensor (20 s).

E_1 is the activation energy of the first reaction (cal/gmole).

E_2 is the activation energy of the second reaction (cal/gmole).

ΔH_1 is the heat of reaction of the first reaction (cal/gmole).

ΔH_2 is the heat of reaction of the second reaction (cal/gmole).

Note that this is a model of a MIMO process. Q_{spec} , F_{spec} , C_{A0} , T_0 and time are independent variables. Q , F , C_A , C_B , C_{Bs} , T , and T_s are dependent variables. V_r , ρ , θ_A , τ_H , τ_v , k_1 , and k_2 are parameters.

3.35 The mole balances on this process should reflect the fact that the product from the first reactor is the feed to the second reactor and that the product of the second reactor is the product of the overall process.

Actuator
$$\frac{dF}{dt} = \frac{1}{\tau_v} (F_{spec} - F)$$

Process
$$V_r \frac{dC_{A1}}{dt} = \frac{F}{\rho} (C_{A0} - C_{A1}) - V_r k C_{A1}^2 \quad (\text{first reactor mole balance on A})$$

$$V_r \frac{dC_{A2}}{dt} = \frac{F}{\rho} (C_{A1} - C_{A2}) - V_r k C_{A2}^2 \quad (\text{mole balance for A on second reactor})$$

Sensors
$$C_{A2s}(t) = C_{A2}(t - \theta_A)$$

where F is the mass feed rate to the reactor (kg/s).

t is time (s).

τ_v is the time constant for the flow control loop on the feed stream (2.0 s).

F_{spec} is the specified mass flow rate to the reactor (kg/s).

V_r is the volume of each stirred tank reactor (l).

C_{A0} is the reactant concentration in the feed to the process (gmole/l).

C_{A1} is the reactant concentration in the first reactor (gmole/l).

C_{A2} is the reactant concentration in the second reactor (gmole/l).

ρ is the density of the reactor feed and product (kg/l).

k is the rate constant for the reaction (l/gmole-s).

C_{A2s} is the measured value of the reactant concentration in the second reactor (gmole/l).

θ_A is the analyzer deadtime for measuring C_{A2} (300 s).

F_{spec} , C_{A0} , and time are the independent variables. F , C_{A1} , C_{A2} , and C_{A2s} are dependent variables. V_r , τ_v , ρ , k , and θ_A are parameters.

3.36 Note that heat is transferred from the steam to the metal of the heat exchanger and from the metal to the process fluid. In addition, since the manipulated variable is steam pressure in the heat exchanger, the correlation between steam pressure and steam temperature must be used:

$$T_{stm} = f(P_{stm})$$

Actuator

$$\frac{dP}{dt} = \frac{1}{\tau_v} (P_{spec} - P)$$

$$T_{stm} = f(P_{stm})$$

Process

$$Q_{stm} = U_{stm} A_{stm} (T_{stm} - T_m)$$

$$Q = UA(T_m - T)$$

$T = 0.5(T_{in} + T_{out}) = T_{in} + \frac{Q}{2FC_p}$ which takes into account that the outlet temperature is

affected by the heat transfer from the metal to the process fluid. Then substituting this result into the equation for Q results in the following after collecting terms and solving for Q .

$$Q = \frac{UA(T_m - T_{in})}{1 + \frac{UA}{2FC_p}}$$

$$M C_{mp} \frac{dT_m}{dt} = Q_{stm} - Q$$

$$T_{out} = T_{in} + \frac{Q}{FC_p} \quad (\text{Output variable})$$

Sensor

$$\frac{dT_s}{dt} = \frac{1}{\tau_{Ts}} (T_{out} - T_s)$$

where P is the pressure of steam in the heat exchanger (atm).

τ_v is the time constant for the flow control loop on the feed stream (2.0 s).

P_{spec} is the specified pressure for steam in the heat exchanger (atm).

T_{stm} is the temperature of steam in the heat exchanger (K).

U is the overall heat transfer coefficient for heat transfer from the metal to the process fluid (cal/m²-sec-K).

U_{stm} is the overall heat transfer coefficient for heat transfer from the metal to the process fluid (cal/m²-sec-K).

A is the area for heat exchange between the metal and the process fluid (m²).

A_{stm} is the area for heat exchange between the steam and the metal (m²).

T_m is the temperature of the metal tubes in the heat exchanger (K).

T is the average temperature of the process fluid in the heat exchanger (K).

T_{out} is the outlet temperature of the process fluid leaving the heat exchanger (K).

T_{in} is the inlet temperature of the process fluid entering the heat exchanger (K).

F is the mass flow rate of the process fluid to the heat exchanger (kg/s).

C_p is the heat capacity of the process fluid (cal/g-K).

M is the mass of the metal tubes in the heat exchanger (kg).

C_{mp} is the heat capacity of the metal tubes in the heat exchanger (cal/g-K).

Q is the rate of heat transfer between the metal tubes and the process fluid (cal/s).

Q_{stm} is the rate of heat transfer between the steam and the metal tubes (cal/s).

τ_{Ts} is the time constant of the temperature sensor (s)

T_s is the measured value of the outlet temperature of the process fluid leaving the heat exchanger (K).

P_{spec} , F , T_{in} and time are independent variables. P , T_{stm} , T_m , T , T_{out} , Q , Q_{stm} , and T_s are dependent variables. U , U_{stm} , τ_v , A , A_{stm} , C_p , C_{mp} , τ_{Ts} , and M are parameters.

3.37 The mole balance for this problem is a little tricky since the volume of the reactor is changing with time. As a result, it is easier to choose the number of moles in the reactor as the state variable and model the volume as a function of time.

Actuator
$$\frac{dF}{dt} = \frac{1}{\tau_v} (F_{spec} - F)$$

Process
$$\frac{dn_A}{dt} = \frac{FC_{A0}}{\rho} - k \left(\frac{n_A}{V_r} \right)^2 V_r$$

$$\frac{dn_B}{dt} = k \left(\frac{n_A}{V_r} \right)^2 V_r$$

$$\frac{dV_r}{dt} = \frac{F(t)}{\rho}$$

$$C_A(t) = \frac{n_A(t)}{V_r(t)}$$

Sensors $C_{As}(t) = C_A(t - \theta_A)$

where F is the mass feed rate to the reactor (kg/s).

t is time (s).

τ_v is the time constant for the flow control loop on the feed stream (2.0 s).

F_{spec} is the specified mass flow rate to the reactor (kg/s).

V_r is the volume of the reactor (l).

C_{A0} is the reactant concentration in the feed to the process (gmoles/l).

n_A is the number of moles of a in the reactor (gmoles).

ρ is the density of the reactor feed and product (kg/l).

k is the rate constant for the first reaction (l/gmole-s).

θ_A is the analyzer deadtime for measuring C_{A2} (300 s).

F_{spec} , C_{A0} , and time are the independent variables. F , C_A , n_A , n_B , V_r , and C_{As} are dependent variables. ρ , k , τ_v and θ_A are parameters.

3.38 Note that the total mass of water and ethyl alcohol in the still decreases as heat is added to the still; therefore, it is easier to model the mass of ethyl alcohol left in the still and the total mass of water and ethyl alcohol left in the still separately. Then the fraction of ethyl alcohol can be calculated by their ratio and the VLE relationship can be used to calculate the composition of the vapor produced.

Actuator
$$\frac{dQ}{dt} = \frac{1}{\tau_H} (Q_{spec} - Q)$$

Process
$$\frac{dM}{dt} = - \frac{Q}{\Delta H_{vap}}$$

$$x(t) = \frac{m_A(t)}{M(t)}$$

$$y(t) = \frac{\alpha x(t)}{1 + (\alpha - 1)x(t)}$$

$$\frac{dm_A}{dt} = - \frac{Q y}{\Delta H_{vap}}$$

Sensor
$$y_s(t) = y(t - \theta_A)$$

where Q is the actual heat addition rate (cal/s).

Q_{spec} is the specified heat addition rate (cal/s).

τ_H is the time constant for the heat addition to the CSTR (20 s).

ΔH_{vap} is the heat of vaporization of the water/alcohol mixture (cal/g).

M is the mass of water and alcohol in the still (g).

x is the mass fraction of alcohol in the still (dimensionless).

y is the mass fraction of alcohol in the vapor leaving the still (dimensionless).

m_A is the mass of alcohol remaining in the still (g).

α is the relative volatility (mass basis) for the alcohol water system.

θ_A is the analyzer deadtime for measuring C_B (30 s).

Note that in a more detailed model of this process the variation of α and ΔH_{vap} with ethyl alcohol concentration would be modeled. Q_{spec} and time are the independent variables. Q , M , x , y , m_A , and y_s are dependent variables. τ_H , ΔH_{vap} , α , and θ_A are parameters.

3.39 In order to more realistically model this process, a dynamic model of the sensor should be used. Otherwise, the system would behave as a pure first order system.

Actuator
$$\frac{dF_{out}}{dt} = \frac{1}{\tau_v} (F_{out,spec} - F_{out})$$

Process
$$\frac{dn}{dt} = F_{in} - F_{out}$$

$$P = \frac{nRT}{V}$$

Sensor
$$P_s = P$$

where F_{out} is the mass flow rate leaving the system (kg/s).

F_{in} is the mass flow rate entering the system (kg/s).

t is time (s).

τ_v is the time constant for the control valve on the exit stream (10 s).

$F_{out,spec}$ is the specified mass flow rate leaving the process (kg/s).

n is the number of moles in the system (kgmoles).

P is the pressure in the system (atm).

T is the temperature in the system (K).

V is the volume of the pressure vessel (m^3).

R is the gas constant ($\text{m}^3\text{-atm/K-kgmole}$).

P_s is the measured pressure in the pressure vessel (atm).

$F_{out,spec}$, F_{in} , T , and time are independent variables. F_{out} , n , P , and P_s are dependent variables. R and τ_v are parameters.

3.40 The model equation for a hot water heater is given by

$$C_p \rho V \frac{dT}{dt} = \rho C_p F (T_{in} - T) + Q$$

Where $V=30$ gal, $\rho=62.4$ lb/ft³, $T(0)=120$ F, $T_{in}=60$, $F=5$ gal/min, and $Q=6 \times 10^6$. After substituting and unit conversions,

$$\frac{dT}{dt} = 840 - 10T$$

After numerical integration,

$$-10 \ln \left[\frac{840 - T}{-360} \right] = t$$

Evaluating this equation for $T=90$ °F yields, $t=7.9$ hours.

3.41 The model equations for this system are

Actuator system
$$\frac{dF_{in}}{dt} = \frac{1}{\tau_v} (F_{in}^{spec} - F_{in})$$

Process model
$$\frac{dM}{dt} = \rho (F_{in} - k\sqrt{H})$$

$$H = \sqrt[3]{\frac{3M}{\pi \rho \tan^2 \theta}}$$

Sensor
$$H_s = H$$

F_{in} is the inlet volumetric flow rate (l^3/s)

F_{in}^{spec} is the setpoint for the flow controller on the inlet feed (l^3/s)

M is the mass of fluid in the cone (g)

ρ is the density of the fluid (g/l)

k is a constant ($\text{l}^3/\text{s-m}^{0.5}$)

H is the height of liquid in the cone (m)

θ is the angle of the cone (degrees)

H_s is the measured height of the fluid level in the tank (m)

The dependent variables are H , H_s , M and F_{in} , and the independent variables are time and F_{in}^{spec} , and the parameters are ρ , θ , and τ_v .

3.42 The fastest dynamic component is the sensor (i.e., $\tau_s=0.1$ s). The slowest dynamics are the control valve (i.e., $\tau_v=10$ s); therefore, from Riggs (1994), the stiffness ratio is given by

$$S = |\lambda_{max}| \Delta t$$

where λ_{max} is the maximum eigenvalue (approximately $1/\tau$) and Δt is the integration time. The integration time is about four times the largest time constant or about 40 in order that the full change in pressure will occur. The maximum eigenvalue is about 10; therefore, S is about 400 which indicates that the problem is somewhat stiff but not excessively stiff. As a result, an Euler integrator can still be used, but an integration step size of 0.05 s is recommended.