Chapter 9

Frequency Response Analysis

Preliminary Questions

9.1 Frequency response analysis is important to the understanding of feedback systems because it shows how the time scale of inputs, such as disturbances, can affect controller performance. For example, if a disturbance is very slow, the feedback controller has ample time to absorb its effect and if a disturbance is relatively fast, the process absorbs its fluctuations before it can affect the controlled variable. On the other hand, disturbances with intermediate frequencies will directly affect the variability from setpoint. Frequency response analysis can also be used to evaluate the propagation of variability from unit to unit through a process. In addition, frequency response analysis contributes a significant number of terms that contribute to the terminology of the process control field.

9.2 A Bode plot is a plot of the logarithm of the amplitude ratio versus the logarithm of the frequency of the input and a plot of the phase angle versus the logarithm of the frequency of the input. For the CST thermal mixer, a Bode plot could be generated by sinusoidally varying the specified flow rate for stream 1 under open loop conditions and measuring the resulting amplitude of the measured temperature of the product after it settles to a fixed standing wave for a range of different frequencies. The amplitude ratio is the ratio of the amplitude in the measured temperature divided by the amplitude of the sinusoidal variation in the specification of the flow rate of stream 1. The phase angle is given by

$$\phi = \frac{\omega \, \Delta t_p}{2\pi} \times 360°$$

where ω is the frequency of the input variations in radians per second and Δt_p is the amount of time that c lags y_s.

9.3 One way to generate a Bode plot using a transfer function of a process is to combine a sinusoidal input with the transfer function directly.

$$Y(s) = G_p(s) \, \frac{a_u \, \omega}{s^2 + \omega^2}$$

Using a partial fraction expansion, take the inverse Laplace transform to form the time domain solution. Then from inspection, the amplitude ratio and phase angle can be identified as a function of frequency.

Another way to use the transfer function to generate a Bode plot is to substitute $s = i\omega$ into $G_p(s)$ and factor the result into the real and imaginary components, i.e.,

$$G_p(i\omega) = R(\omega) + iI(\omega)$$

then
$$A_r = \left|G_p(i\omega)\right| = \sqrt{R^2(\omega) + I^2(\omega)}$$

and
$$\phi = \tan^{-1}\,[I(\omega)/R(\omega)]$$

From these equations A_r and ϕ can be plotted versus ω yielding a Bode plot.

9.4 The amplitude ratio used in the Bode plot is A_r where

$$A_r = \frac{a_y}{a_c}$$

and a_y is the amplitude of the sinusoidal variation in the controlled variable, y, and a_c is the sinusoidal variation in the controller output, c.

9.5 The phase angle used in a Bode plot is ϕ in degrees where

$$\phi = \frac{\omega \Delta t_p}{2\pi} 360°$$

and ω is the frequency of the input variations in radians per second and Δt_p is the time difference between peaks of the controller output and the controlled variable.

9.6 A Bode plot for a system is a plot of the logarithm of the amplitude ratio and a linear scale for the phase angle versus the logarithm of the frequency.

9.7 The amplitude of a sequence of transfer functions is the product of the amplitude ratio of the transfer functions.

9.8 The phase angle of a sequence of transfer functions is the sum of the phase angles of the transfer functions.

9.9 When the amplitude ratio of the open loop transfer function including the controller exceeds unity at the crossover frequency, then the closed loop transfer function will be unstable. At the crossover frequency (ϕ = -180º), the feedback of the measurement back into the controller becomes unstable when the amplitude ratio is loop Bode plots of several processes in series, one can evaluate whether or not variabilities will pass through the system of controllers.

9.16 The peak frequency of a controller indicates the frequency of a disturbance that has the largest effect on the control performance.

9.17 The frequency of the disturbance input is varied to generate a closed-loop Bode plot.

9.18 The closed-loop Bode plot of a process indicates the disturbance frequencies for which the controller is most sensitive. This information can be used to analyze how disturbances dampen or propagate from one control loop to another through a sequence of processes.

9.19 Bode plots can be generated by

(1) directly exciting the process with sinusoidal inputs

(2) applying a sinusoidal input to the transfer function of the process

(3) substituting $s = i\omega$ into the transfer function and rearranging

(4) applying a pulse test

Analytical Questions and Exercises

9.20 (a) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{0.4}{5\omega}\sqrt{\frac{25\omega^2 + 1}{36\omega^2 + 1}}$$

$$\phi = \tan^{-1}\left(\frac{-1}{5\omega}\right) + \tan(-6\omega) - 360\omega / 2\pi$$

(b) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{2}{\sqrt{(1-9\omega^2)^2 + 144\omega^2}}$$

$$\phi = \tan^{-1}\left(\frac{-12\omega}{1-9\omega^2}\right) - 540\omega / 2\pi$$

(c) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{3}{5\omega^2}\sqrt{25\omega^2 + 1}$$
$$\phi = \tan^{-1}\left(\frac{-1}{5\omega}\right) - 90$$

(d) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{3}{5\omega^2}\sqrt{25\omega^2 + 1}$$
$$\phi = \tan^{-1}\left(\frac{-1}{5\omega}\right) - 90 - 360\omega / 2\pi$$

(e) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{1.3}{\sqrt{(1 - 900\omega^2)^2 + 240^2\omega^2}}\sqrt{1 + \frac{1}{100\omega^2}}$$
$$\phi = \tan^{-1}\left(\frac{-240\omega}{1 - 900\omega^2}\right) + \tan^{-1}\left(\frac{-1}{10\omega}\right) - (15)(360)\omega / 2\pi$$

(f) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{6}{\omega}\sqrt{\frac{\omega^2 + 1}{16\omega^2 + 1}}$$
$$\phi = \tan^{-1}\left(\frac{-1}{\omega}\right) + \tan^{-1}(-4\omega) - 720\omega / 2\pi$$

(g) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{69}{15\omega^2}\sqrt{225\omega^2 + 1}$$
$$\phi = \tan^{-1}\left(\frac{-1}{15\omega}\right) - 90 - 1080\omega / 2\pi$$

(h) Using Table 9.1 with the parameter specifications yields

$$A_r = \frac{5}{\sqrt{(1 - 900\omega^2)^2 + 120^2\omega^2}}$$
$$\phi = \tan^{-1}\left(\frac{-120\omega}{1 - 900\omega^2}\right) - 3600\omega / 2\pi$$

9.21 (a) Using Table 9.1 in a manner similar to Examples 9.1 and 9.2, the amplitude ratio and the phase angle can be expressed by