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Georgia Tech
ChE 4400, Spring 2003
Midterm Exam II, April 1, 2003
Closed Book, Closed Notes
80 minutes

1. **(2pts)** Which of the following is true about PID tuning?
   a. A relay is the preferred method for generating the reaction curve.
   b. Continuous cycling yields accurate information on the frequency response of the plant at the critical frequency.
   c. Reaction curve based fit generally yields a model with an accurate frequency response around the critical frequency.
   d. Continuous cycling yields parameters that can be used with IAE/ISE/ITAE tuning rules.

2. **(2pts)** Which of the following strategies are NOT used simultaneously with feedback control?
   a. Feedforward control
   b. Cascade control
   c. Relay based control
   d. Smith Predictor

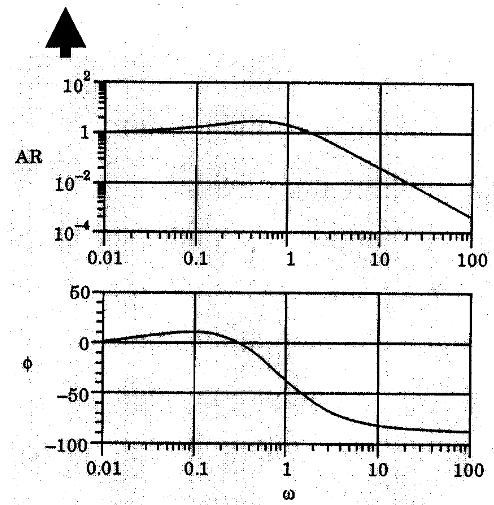
3. **(2pts)** Which of the following is true about Smith Predictor?
   a. It is designed to reduce interaction.
   b. It removes the time delay, thereby giving a delay-free response.
   c. It effectively takes the time delay out of the loop, thereby simplifying the PID tuning and improving the performance.
   d. It increases robustness, especially with respect to errors in the time delay estimation.

4. **(2pts)** Which of the following is true about cascade control?
   a. To be effective, the inner-loop process must be significantly slower than the outer-loop process.
   b. It is particularly effective for controlling a flowrate precisely, with a valve positioner as the inner loop.
   c. It can be used to remove disturbances in the steam or coolant temperature in a jacketed reactor before they affect the reactor temperature.
   d. It increases the effect of valve nonlinearity even though it speeds up the valve response.

5. **(2pts)** Which of the following is true about the parameter τ_C (or λ) in IMC tuning?
   a. It's a parameter that one obtains from a reaction curve.
   b. Smaller τ_C means more robustness meaning the controller can tolerate larger model / plant mismatch.

c. Smaller τ_C means more noise filtering.
d. Smaller τ_C means faster response.

6. **(2pts)** What is the distinct feature in the step response curve of the system that the Bode diagram on the left-hand side below represents?
   a. Time delay b. Inverse response c. Overshoot d. Oscillation



7. **(2pts)** Which of the following system has no limit on the size of controller gain (for P-only control) one can apply before making the system unstable? (Hint: Use the Bode stability criterion.)

   a. $\dfrac{5}{(2s+1)(3s+1)(10s+1)}$ b. $\dfrac{5(-10s+1)}{(2s+1)(3s+1)}$

   c. $\dfrac{5(10s+1)}{(2s+1)(3s+1)}$ d. $\dfrac{1}{(2s+1)}e^{-0.5s}$

8. **(2pts)** Which of the following is true about Fourier transform?
   a. It can be applied to sinusoidal signals only.
   b. The Fourier transform and its inverse can be implemented efficiently through a numerical algorithm.
   c. Inverse transform requires Partial Fraction Expansion just as in Laplace transform.
   d. A real disadvantage over Laplace transform is that the parameter ω has no physical meaning.

9. **(2pts)** The system $\dfrac{100}{(10s+1)(10s+1)(10s+1)}e^{-10s}$ is subjected to a sinusoidal forcing of frequency 5 rad/time and amplitude of 3. The output response is also

sinusoidal at the same frequency of 5 rad/min. What is the amplitude of the output response?

a. 0.0008 b. 0.3 c. 0.0024 d. 0.1

Hint: AR of $\frac{1}{\tau s+1}$ is $\frac{1}{\sqrt{\tau^2\omega^2+1}}$.

10. **(2pts)** Suppose the ultimate gain of the process was found to be 5. The P-only controller gain value that gives the gain margin of 2 is?

a. 5 b. 3.33 c. 2.5 d. 1.5

9. $AR = \frac{k}{\sqrt{\tau_1^2\omega^2+1}\sqrt{\tau_2^2\omega^2+1}*\sqrt{\tau_3^2\omega^2+1}} = \frac{100}{\left[\sqrt{(10\times5)^2+1}\,\right]^3} \approx \frac{100}{50^3} = \frac{1}{1250} = 8\times10^{-4}$

$[\text{Amplitude}]_{o/p} = [\text{Amplitude}]_{i/p} * AR = 2.4\times10^{-3}$

10. $K_u = 5 \Rightarrow [AR_{sys}]_{\omega_c} = \frac{1}{5}$

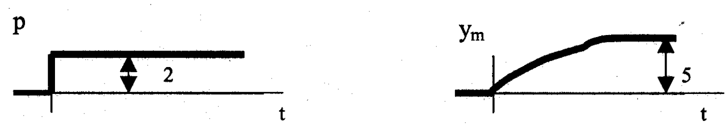
$GM = \frac{1}{[AR_{sys}]_{\omega_c} * K_c} \Rightarrow 2 = \frac{1}{(1/5)K_c} \Rightarrow K_c = 2.5$

11. **(10pts)** Your professor has the brilliant idea that one would combine the result of step test and continuous cycling to build a first order plus time delay model. Shown below is the result of a step test. From the continuous cycling experiment, the ultimate period was found to be 5 minutes. The controller gain that induced continuous cycling is 3.

    a. **(2pts)** Calculate the process gain from the step test result.
    b. **(3pts)** From the result of the continuous cycling experiment, what are the critical frequency ω_c and the amplitude ratio of the plant G at that frequency?
    c. **(5pts)** Calculate the time constant and the time delay that are consistent with the result of the continuous cycling experiment.

Hint: The A.R. for FOPTD system is $1/\sqrt{1+\tau^2\omega^2}$

The phase angle for FOPTD system is $\tan^{-1}(-\tau\omega) - \theta\omega$



11a. $\text{Gain} = \dfrac{[\Delta\,\text{output}]_{ss}}{[\Delta\,\text{input}]_{ss}} = \dfrac{5}{2} = 2.5$

11b. $P_u = \dfrac{2\pi}{\omega_c} \Rightarrow \omega_c = \dfrac{2\pi}{5} = 1.256 \text{ rad/min}$

$[AR_{sys}]_{\omega_c} = \dfrac{1}{K_u} \Rightarrow [AR_{sys}]_{\omega_c} = 0.33$

11c. $G = \dfrac{k}{\tau s + 1} e^{-\theta s} \xrightarrow[(AR)]{\text{At } \omega_c} AR = \dfrac{k}{\sqrt{\tau^2\omega_c^2+1}} \Rightarrow \dfrac{1}{3} = \dfrac{2.5}{\sqrt{\tau^2\omega_c^2+1}}$

$\Rightarrow \sqrt{\tau^2\omega_c^2+1} = 3 * 2.5$

$\Rightarrow \tau^2\omega_c^2 + 1 = (7.5)^2 = 56.25$

$\Rightarrow \boxed{\tau = \dfrac{\sqrt{55.25}}{\omega_c} = 5.915 \text{ min.}}$

$\Big\downarrow$ At ω_c (ϕ)

$\phi = \text{Tan}^{-1}[-\omega_c\tau] - \theta\omega_c$

[Since ω_c is in rad/min, use $\phi = -\pi$ AND NOT $\phi = -180°$]

$-\pi = \text{Tan}^{-1}[-\sqrt{55.25}] - \theta\omega_c$

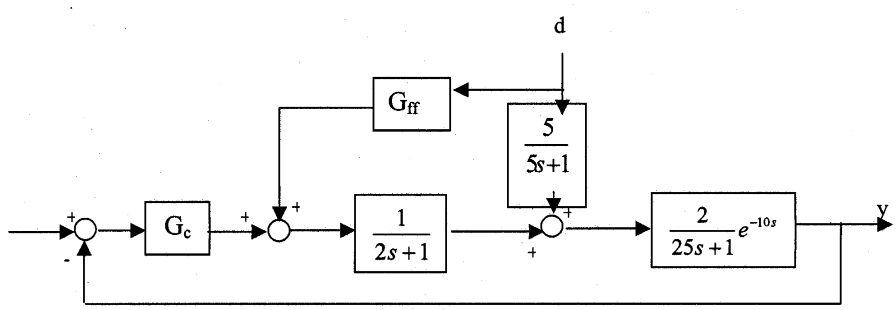
$\Rightarrow \left\{\theta = \dfrac{1}{\omega_c}\left[\text{Tan}^{-1}(-\sqrt{55.25}) + \pi\right] = 1.356 \text{ minutes}\right\}$

12. **(20pts)** Sketch the shape of the (unit-size) step response of the following transfer functions. Do indicate the starting and final settling value, if the system is stable. Also, sketch the Bode diagrams. For phase angle plots, clearly indicate the starting and final values.

   a. $1/s$
   b. $(1+1/s)$
   c. $5(10s+1)/(s+1)(0.1s+1)$
   d. $5(0.1s+1)/(s+1)(10s+1)$
   e. $5(-10s+1)/(0.1s+1)(s+1)$
   f. $2/(s^2+0.01s+1)$
   g. $2/(s^2-0.01s+1)$
   h. $2/(s^2+7s+10)$
   i. $3e^{-3s}$
   j. $4/(5s+1)(0.01s+1)^{100}$

[SEE ATTACHED SHEETS]

13. **(10pts)** Explain the following control strategy.
   a. **(4pts)** Derive the closed-loop transfer function from d to y.
   b. **(3pts)** Design the feedforward controller G_{ff} to yield an ideal response.
   c. **(3pts)** Would feedforward controller in addition to the feedback controller be useful in this case? Why or why not?



13(a) $\text{Num} = \left[G_{ff}\left[\frac{1}{2s+1}\right] + \frac{5}{5s+1}\right]\left[\frac{2e^{-10s}}{25s+1}\right]$

$\text{Den} = 1 + G_c\left[\frac{1}{2s+1}\right]\left[\frac{2e^{-10s}}{25s+1}\right]$

$$\therefore \frac{y(s)}{d(s)} = \frac{2\left[(5s+1)G_{ff} + 5(2s+1)\right]e^{-10s} \Big/ \cancel{(2s+1)}(5s+1)\cancel{(25s+1)}}{\left[(2s+1)(25s+1) + 2G_c e^{-10s}\right] \Big/ \cancel{(2s+1)}\cancel{(25s+1)}}$$

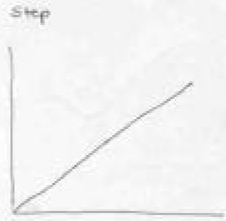
$$\boxed{\frac{y(s)}{d(s)} = \frac{2\left[G_{ff}(5s+1) + 5(2s+1)\right]e^{-10s}}{\left[(2s+1)(25s+1) + 2G_c e^{-10s}\right](5s+1)}}$$

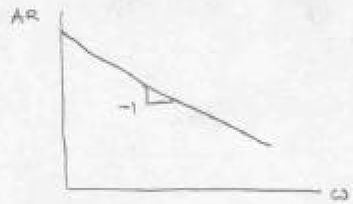
13(b) Ideal Response $\Rightarrow$ Numerator = 0

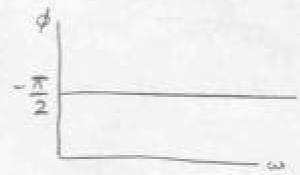
$$\boxed{\therefore G_{ff} = -\frac{5(2s+1)}{5s+1}}$$

13(c) YES IT IS USEFUL because the process has significant DELAY (e^{-10s}) and SLOW DYNAMICS ($\tau = 25$). Hence F.F. enables compensation of disturbance before it affects the process.

12(a) $\frac{1}{s}$

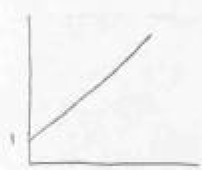


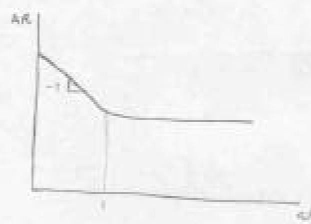


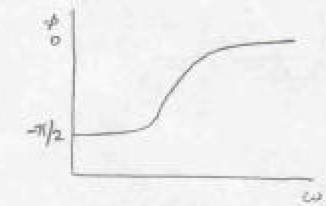


12(b) $1 + \frac{1}{s}$

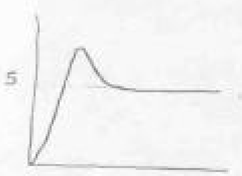
$f(t) = 1 + t$

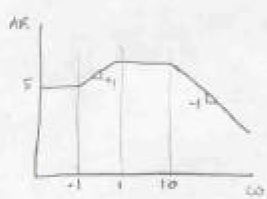


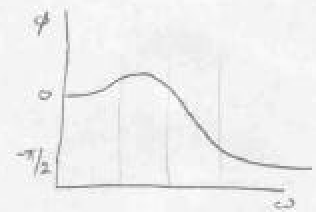




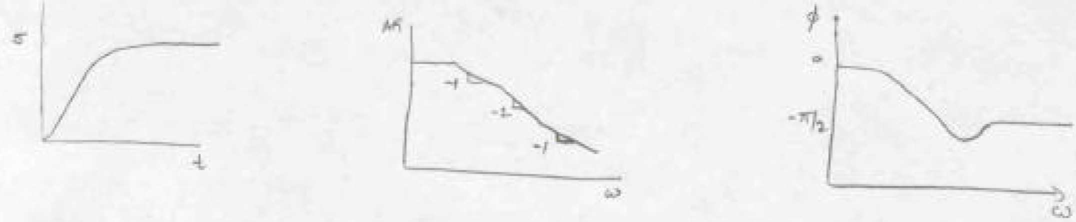
12(c) $\frac{5(10s+1)}{(s+1)(0.1s+1)}$



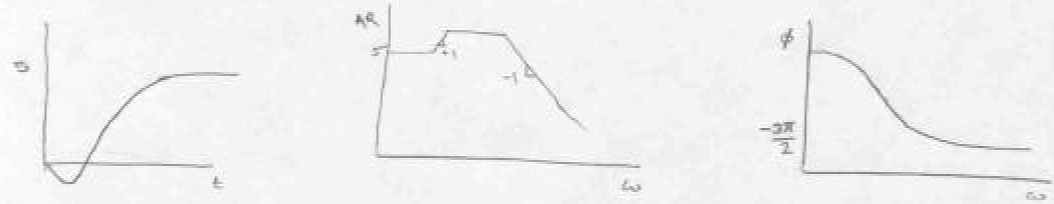




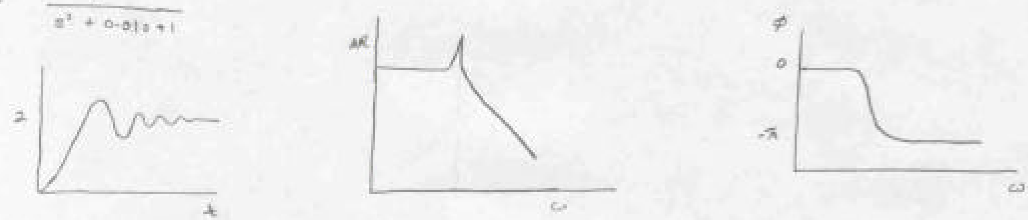
12(d) $\dfrac{5(0.1s+1)}{(s+1)(10s+1)}$



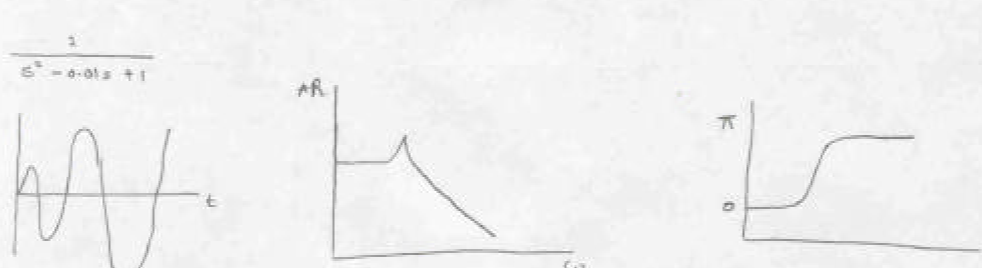
12(e) $\dfrac{5(-10s+1)}{(0.1s+1)(s+1)}$



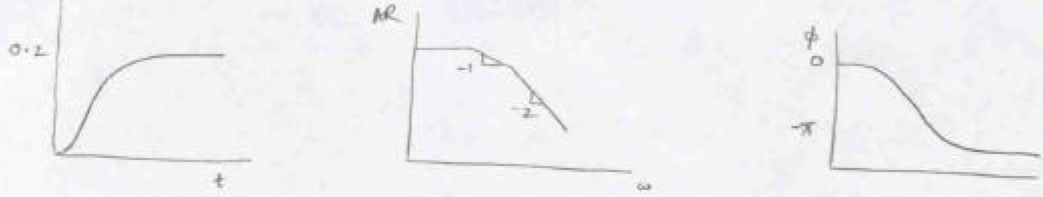
12(f) $\dfrac{2}{s^2+0.01s+1}$



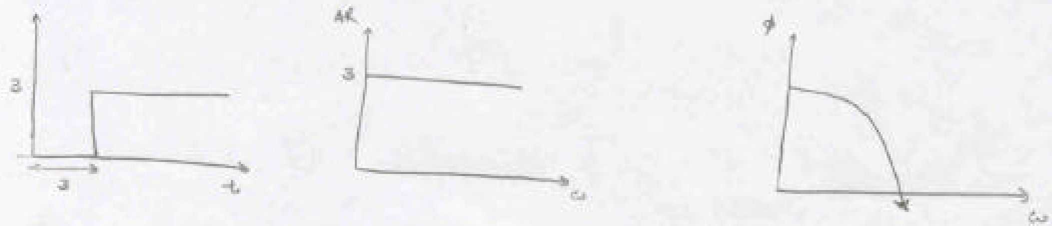
12(g) $\dfrac{2}{s^2-0.01s+1}$



12(h) $\frac{2}{s^2+7s+10}$



12 (i) $3e^{-3s}$



12(j) $\frac{4}{(5s+1)(0.01s+1)^{100}}$

$\approx \frac{4}{(5s+1)} e^{-1s}$