Solutions to Quiz I

Note: Part (e) will be considered as an extra credit problem. The score of this test is out of 8 points, which will be scaled accordingly.

Problem 1 DP Cell Problem

Problem 1-a Sizing of the cell

Here, you are asked to size the orifice such that a maximum flow rate of 100 gal/min can be measured using a DP cell having range 0-5 psi. Hence, for this problem, we have

$$\Delta P_{max} = 5psi \qquad Q_{max} = 100gpm$$ 0.5 points

Substituting values in the orifice equation provided (**use appropriate units**), we get

$$\begin{aligned} \frac{100\ gal}{min}\left|\frac{1\ min}{60\ sec}\right|\frac{1\ ft^3}{7.481\ gal}\left|\frac{[12\ in]^3}{[1\ ft]^3}\right. &= \frac{0.6A_2}{\sqrt{1-\frac{A_2^2}{7.393^2}}}\sqrt{2\frac{32.2\ lb_m\ ft}{lb_f\ sec^2}\left|\frac{ft^3}{62.4\ lb_m}\right|\frac{5\ lb_f}{in^2}\left|\frac{[12\ in]^4}{ft^4}\right.} \\ 385.0\frac{in^3}{sec} &= \frac{0.6A_2}{\sqrt{1-\frac{A_2^2}{7.393^2}}}\cdot 327.11\frac{in}{sec} \end{aligned}$$

1 point

0.5 points

Some of you chose to express the solution in ft instead of in. In these cases, the left hand side term becomes $0.223\ ft^3/sec$ and the last term or right hand side becomes $27.26\ ft/sec$. In this case, please be sure to use $A_1 = 0.0513\ ft^2$ instead.

If you do not have a calculator, writing "this can be solved using a calculator/Excel" will not yield you credit. In such a case, the first step above needs to be done correctly. Next, you may represent

$$P = \frac{100\ gal}{min}\left|\frac{1\ min}{60\ sec}\right|\frac{1\ ft^3}{7.481\ gal}\left|\frac{[12\ in]^3}{[1\ ft]^3}\right.$$

and the term in square root as Q. Finally, obtain the solution in terms of variables P, Q and known quantities.

OK, continuing with the solution, we get

$$1.9616\ in^2\sqrt{1-\frac{A_2^2}{7.393^2}} = A_2$$

Squaring both sides, we get

$$\begin{aligned} 3.848\left[1-\frac{A_2^2}{54.656}\right] &= A_2^2 \\ 3.848 &= A_2^2\left[1+\frac{3.848}{54.656}\right] \\ \text{Hence, } A_2 = 1.896\ in^2 &= 0.0132\ ft^2 \end{aligned}$$

1 point

Problem 1-b

$\beta = A_2/A_1 = 0.256$, which lies in the range 0.2 to 0.7. Also, maximum pressure drop across the DP cell is 3.33% of the total line pressure. Hence the design is **acceptable**.

1 point for each argument
(Total = 2 points)

Problem 1-c Electric signal for square root extraction

For this problem, you don't need to solve the entire expression. For a specific design of DP

$$Q_1 = \frac{C_d A_2}{\sqrt{1-\beta^2}}\sqrt{\frac{2g_c}{\rho}}\sqrt{\Delta P_1}$$

$$Q_2 = \frac{C_d A_2}{\sqrt{1-\beta^2}}\sqrt{\frac{2g_c}{\rho}}\sqrt{\Delta P_2}$$

$$\text{Hence, } \frac{Q_1}{Q_2} = \sqrt{\frac{\Delta P_1}{\Delta P_2}}$$

1 point

For $Q_1 = 100\ gpm$, $\Delta P = 5\ psi$. Then, for $Q_2 = 80\ gpm$,

$$\Delta P_2 = 5 \times (80/100)^2 = 3.2\ psi$$

For square root extraction, the signal is proportional to the square root of pressure drop. The range 4-20 mA corresponds to 0-5 psi. Thus,

$$mA = 4 + \frac{16}{\sqrt{5}}\sqrt{\Delta P} = 4 + 16 \times \sqrt{\frac{3.2}{5}}$$

1 point

Hence the signal will be 16.8 mA.

Problem 1-d Square root extraction is not used

In this case, the signal is proportional to the pressure drop. Hence, as above, the signal will be $4 + 16 \times \frac{3.2}{5}$, which is 14.24 mA.

1 point

These two parts of the problem can also be solved in a simpler way. We realize that flow rate is proportional to the square root of pressure drop. For square root extraction, electric signal is also proportional to the square root of pressure drop. Hence, signal will be directly proportional to the flow rate; i.e. signal= $4 + 16 \times \frac{Q}{100}$.

Similarly, if square root extraction is not used, signal is proportional to the pressure drop, which in turn is proportional to the square of the flow rate ($Q \propto \sqrt{\Delta P} \Rightarrow \Delta P \propto Q^2$). Hence, for this case, signal= $4 + 16 \times \frac{Q^2}{100^2}$.

Problem 1-e Undersized orifice

Now, we have $A_2^{new} = 0.8A_2$. Also, A_1 remains unchanged and $\beta^{new} = 0.8\beta$. Thus, we have

$$\frac{Q^{new}}{Q} = \frac{A_2^{new}}{\sqrt{1-[\beta^{new}]^2}} \frac{\sqrt{1-\beta^2}}{A_2}$$

Extra credit: 2 points

Solving, we obtain $Q^{new} = 79\ gal/min$.